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# Developments in the Control Loops Benchmarking

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## 1. Introduction

The problem of control performance assessment and monitoring is getting more and more significant because control systems have much bigger influence on accomplishing aims determined by companies. These aims mean achieving goals connected with quality, safety and profits. This justifies academic and commercial interest in development of methods for analyzing the quality of such systems which allow to avoid unreliable human factor. The control system performance cannot be depicted by means of only one simple statistics. The whole procedure called control loops benchmarking (Harris & Seppala 2002) requires much more complicated multi-stage process consisting of: data acquisition, analysis and diagnostics (making the tool based on mathematical model of process, the lower bound estimation, the existing control loop performance assessment, testing of the performance improvement using existing controller structure), retuning or control algorithm replacement.

## 2. Background of the problem

Complex systems usually are comprised of numerous loops which are controlled by local SISO controllers. Most of these industrial control loops are equipped with PID type controllers whose parameters are usually tuned using classical approach that neglects the disturbance characteristics. The decision to retune or replace any of these controllers should be preceded by an investigation whether and to what extent this would improve performance. Such procedure is referred to benchmarking or control performance assessment (Desborough & Harris 1992). Most of research done so far assumes MV (minimum variance) control as the performance lower bound and variance of the system output as basic quantity for control quality assessment. The classical (Huang 2003) performance measure is as follows

$$\eta = \frac{\sigma_{mv}^2}{\sigma_y^2}, \quad (1)$$

where the hypothetical minimum variance,  $\sigma_{mv}^2$  is determined analytically by

$$\sigma_{mv}^2 = (f_0^2 + f_1^2 + f_2^2 + \dots + f_{l-1}^2) \sigma_a^2 \quad (2)$$

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and defined by impulse response coefficients  $f_0, f_1, \dots, f_{l-1}$ , variance  $\sigma_a^2$  of the disturbance model driving noise, and discrete-time delay  $l$ . It is very important that  $\eta$  can be estimated directly from loop operating data. The exact value of delay  $l$  is assumed to be known to the process engineer. Unfortunately, very often there is no pure delay in the process, and the value of delay  $\tau$  of the frequently used lag-delay model

$$H(s) = \frac{l}{Ts + 1} e^{-s\tau} \quad (3)$$

is then used to determine  $l$  given sampling period  $h$ . As shown in (Błachuta & Bialic 2005), in this case the value of index  $\eta$  may be not very bad even in a relatively purely tuned control system hiding the fact that the best achievable accuracy referring to  $l=1$  can be much better. Moreover MV based benchmark does not take the control effort into account and because of large magnitudes of control signal it is often useless.

In this respect, the modified MV control strategy with bounded control variance is used as benchmark in the chapter. This results in the LQG control algorithm allowing control performance assessment under assumption of the same control effort.

### 3. Control problem statement

It is assumed that the linear SISO plant is modeled by the following stochastic, continuous-time system

$$\frac{dx(t)}{dt} = Ax(t) + bu(t - \tau) + c\dot{\xi}(t) \quad (4)$$

$$y(t) = d'x(t) \quad (5)$$

where  $x(t)$  is  $p$  - dimensional state vector,  $A$  is  $p \times p$  - dimensional matrix,  $b, c$  and  $d$  are  $p$  - dimensional vectors. The initial condition  $x_0$  is assumed to be a normal random vector,  $x_0 \sim N(0, Q_0)$ .  $\xi(t)$  is a Wiener process, and  $\text{var } \xi(t) = \delta(t)$ . The time delay is defined as follows:

$$\tau = lh - h + \theta \quad (6)$$

where  $l \geq 1$  and  $0 < \theta \leq h$ . The plant is controlled by the output  $u(t)$  of a ZOH device with period  $h$

$$u(t) = u_k, \text{ for } t \in (kh, kh + h], \quad k = 0, 1, \dots, \quad (7)$$

driven by the digital controller output  $u(k)$ , which changes its values at discrete time instants  $t_k = kh, k = 0, 1, \dots$ . The output of the system is assumed to be measured synchronically at instants  $t_k$  as:

$$y_k = d'x_k + n_k \quad (8)$$

where  $n_k$  is measurement error composed of white noise with zero mean  $E[n_k] = 0$ , and variance  $E[n_k^2] = v^2$ . The variance of measurement noise characterizes accuracy of the sensor, transmitter and A/D converter.

The aim of the system is to minimize the average value of the system error variance with the control variance limit. The considered problem is equivalent to minimization of the weighted  $H_2$  performance index:

$$I = \lim_{N \rightarrow \infty} E \frac{1}{Nh} \int_0^{Nh} \{ y^2(t + \tau) + \lambda u^2(t) \} dt \quad (9)$$

This index with controlled output  $y(t)$  and control signal  $u(t)$  is minimized both for optimal unrestricted LQG and classical restricted structure PID controllers (Grimble 2003) such that the maximum efficiency in terms of disturbance attenuation is achieved under bounded control variance.

## 4. Control algorithms

### 4.1 LQG benchmark

Introduce the predictable state

$$x^p(t) = x(t + \tau) \quad (10)$$

The system defined by state equation (4), measurement equation (8), modulation equation (7) and performance index (9) can be described at sampling instants as

$$x_{k+1}^p = Fx_k^p + gu_k + w_k^p \quad (11)$$

$$z_k = d' x_k^p(kh - \tau) + n_k \quad (12)$$

$$I = \lim_{N \rightarrow \infty} E \frac{1}{N} \sum_{k=0}^{N-1} \{ x_k^{p'} Q_1 x_k^p + 2x_k^{p'} q_{12} u_k + q_2 u_k^2 + q_w \} \quad (13)$$

where  $w_k^p = w(kh + \tau)$  is zero mean Gaussian white noise vector with covariance  $E\{w_k^p, w_k^p'\}$ . Vectors  $x_0$  and  $[w_k^p, n_k]$  are uncorrelated for all  $k \geq 0$ . Matrices defining the system (11) -(12) are as follows

$$\left. \begin{aligned} F(h) &= e^{Ah}, & w(h) &= \int_0^h e^{A(h-s)} c \xi(s) ds \\ g(h) &= \int_0^h e^{Av} b dv, & W(h) &= \int_0^h e^{As} c c' e^{A's} ds \end{aligned} \right\} \quad (14)$$

and matrices defining performance criterion (13) are as follows

$$\left. \begin{aligned} Q_1 &= \frac{1}{h} \int_0^h F'(\tau) M F(\tau) d\tau, \\ q_{12} &= \frac{1}{h} \int_0^h F'(\tau) M g(\tau) d\tau, \\ q_2 &= \frac{1}{h} \int_0^h g'(\tau) M g(\tau) d\tau, \\ q_w &= \frac{1}{h} d' \left\{ \int_0^h \int_0^\tau F(\tau-s) c c' F'(\tau-s) ds d\tau \right\} d \end{aligned} \right\} \quad (15)$$

The optimal control law minimizing the performance index (13) for the system (11)-(12) is

$$u_k = -k_c' \hat{x}_{k|k}^p = -k_c' F_p \underbrace{\begin{bmatrix} \hat{x}_{k|k} \\ u_{k-d} \\ \vdots \\ u_{k-2} \\ u_{k-1} \end{bmatrix}}_{\hat{x}_{k|k}^p} = -k_c' F_p \hat{x}_{k|k} \quad (16)$$

where

$$F_p = [F(\tau) \quad F(\tau - \theta)g(\theta) \quad F(lh - 2h)g \quad \cdots \quad Fg \quad g] \quad (17)$$

$\hat{x}_{k|k}^p$  is an estimate of the state  $x_k^p$  using measurements  $z_k$  up to and including  $k$ . The feedback gain vector depends on the positive solution  $S$  of the following algebraic Riccati equation

$$S = Q_1 + F'SF - \frac{(q_{12} + F'Sg)(q_{12} + F'Sg)'}{q_2 + g'Sg} \quad (18)$$

$$k_c = \frac{q_{12} + F'Sg}{q_2 + g'Sg} \quad (19)$$

Stationary Kalman filter (Åström & Wittenmark 1990) for the system (11)-(12) takes the following form

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + k^f (z_k - d' \hat{x}_{k|k-1}) \quad (20)$$

$$\hat{x}_{k+1|k} = F\hat{x}_{k|k} + g(h - \theta)u_{k-l+1} + F(h - \theta)g(\theta)u_{k-l} \quad (21)$$

where

$$k^f = \frac{\Sigma d}{v^2 + d' \Sigma d} \quad (22)$$

$$\Sigma = W + F \left( \Sigma - \frac{\Sigma d d' \Sigma'}{v^2 + d' \Sigma d} \right) F' \quad (23)$$

#### 4.2 PID type controllers

The system (4)-(5) controlled by means of discrete time classical PID type controllers is considered. Controller settings are supplied in two ways: as minimization result of the performance index (13), and by means of one of the classical methods called QDR (Quarter Decay Ratio).

The control law for classical controllers is defined by the following equations -state equation

$$x_{k+1}^c = A^c x_k^c - B^c z_k \quad (24)$$

- and output equation

$$u_k = C^c x_k^c - D^c z_k, \quad (25)$$

where for discrete time PID controller matrices and vectors defining the control law are the following form

$$\left. \begin{aligned} A^c &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, & C^c &= \begin{bmatrix} K_p \frac{h}{T_i} & -K_p \frac{T_d}{h} \end{bmatrix}, \\ B^c &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}, & D^c &= \begin{bmatrix} K_p \left( 1 + \frac{h}{T_i} + \frac{T_d}{h} \right) \end{bmatrix}, \end{aligned} \right\} \quad (26)$$

Since the predictable description of the system (11)-(12) is not useful for the PID control algorithm, the system defined by state equation (4), measurement equation (8), modulation equation (7) and performance index (9) is described at sampling instants in alternative form

$$x_{k+1} = Fx_k + \Gamma_0 u_{k-l+1} + \Gamma_1 u_{k-l} + w_k \quad (27)$$

$$z_k = d' x_k + n_k \quad (28)$$

where

$$\Gamma_1 = e^{A(h-\theta)} \int_0^\theta e^{Av} b dv = F(h-\theta)g(\theta), \quad \Gamma_0 = \int_0^{h-\theta} e^{Av} b dv = g(h-\theta) \quad (29)$$

The corresponding state-space description is as follows

$$\begin{bmatrix} x_{k+1} \\ u_{k-l+1} \\ \vdots \\ u_{k-1} \\ u_k \end{bmatrix} = \underbrace{\begin{bmatrix} F & \Gamma_1 & \Gamma_0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}}_{\underline{F}} \underbrace{\begin{bmatrix} x_k \\ u_{k-l} \\ \vdots \\ u_{k-2} \\ u_{k-1} \end{bmatrix}}_{\underline{x}_k} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}}_{\underline{\Gamma}} u_k + \underbrace{\begin{bmatrix} I \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{\underline{\Omega}} w_k \quad (30)$$

Introduce the notation

$$\left. \begin{aligned} \bar{P} &= E\{\bar{x}_k, \bar{x}_k'\} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix} \\ \bar{x}_k &= \begin{bmatrix} \underline{x}_k \\ x_k^c \end{bmatrix}, \quad \underline{x}_k = \begin{bmatrix} x_k \\ u_{k-l} \\ \vdots \\ u_{k-2} \\ u_{k-1} \end{bmatrix} \end{aligned} \right\} \quad (31)$$

Employing (31) and (25), the performance index (13) can be rewritten as

$$I = \left. \begin{aligned} & \text{tr}\{Q_1 F_p \bar{P}_{11} F_p' + Q_1 W(\tau)\} - \text{tr}\{F_p' q_{12} D^c \underline{d}' \bar{P}_{11}\} \\ & - \text{tr}\{\underline{d} D^c q_{12}' F_p \bar{P}_{11}\} + \text{tr}\{q_2 \underline{d} D^c D^c \underline{d}' \bar{P}_{11}\} \\ & + \text{tr}\{C^c q_{12}' F_p \bar{P}_{12}\} - \text{tr}\{q_2 C^c D^c \underline{d}' \bar{P}_{12}\} \\ & + \text{tr}\{F_p q_{12} C^c \bar{P}_{21}\} - \text{tr}\{q_2 \underline{d} D^c C^c \bar{P}_{21}\} \\ & + \text{tr}\{q_2 C^c C^c \bar{P}_{22}\} + \text{tr}\{q_2 D^c D^c v^2 + q_w\} \end{aligned} \right\} \quad (32)$$

### 4.3 Output and control variances

The output and control variances at sampling instants for the LQG controller can be calculated from the following expressions

$$\sigma_{y_k LQG}^2 = \underline{d}' E\{\underline{x}_k, \underline{x}_k'\} \underline{d} = \underline{d}' \bar{X}(11) \underline{d} \quad (33)$$

$$\sigma_{u_k LQG}^2 = k^c F_p E\{\hat{\underline{x}}_{k|k}, \hat{\underline{x}}_{k|k}'\} F_p' k^c = k^c F_p \bar{X}(22) F_p' k^c \quad (34)$$

where

$$\bar{X} = E\{\bar{\underline{x}}_k, \bar{\underline{x}}_k'\} = \left. \begin{aligned} & \begin{bmatrix} \bar{X}(11) & \bar{X}(12) \\ \bar{X}(21) & \bar{X}(22) \end{bmatrix} \\ & \bar{\underline{x}}_k = \begin{bmatrix} \underline{x}_k \\ \hat{\underline{x}}_{k|k} \end{bmatrix}, \quad \underline{x}_k = \begin{bmatrix} \underline{x}_k \\ u_{k-1} \\ \vdots \\ u_{k-2} \\ u_{k-1} \end{bmatrix}, \quad \hat{\underline{x}}_k = \begin{bmatrix} \hat{\underline{x}}_{k|k} \\ u_{k-1} \\ \vdots \\ u_{k-2} \\ u_{k-1} \end{bmatrix} \end{aligned} \right\} \quad (35)$$

and formulas in case of PID type controllers are defined as follows

$$\sigma_{y_k PID}^2 = \text{var}\{y_k\} = \underline{d}' \bar{P}_{11} \underline{d} \quad (36)$$

$$\sigma_{u_k PID}^2 = \text{var}\{u_k\} = \left. \begin{aligned} & C^c \bar{P}_{22} C^{c'} - D^c \underline{d}' \bar{P}_{12} C^{c'} \\ & - C^c \bar{P}_{21} \underline{d} D^{c'} + D^c \underline{d}' \bar{P}_{11} \underline{d} D^{c'} + D^c v^2 D^{c'} \end{aligned} \right\} \quad (37)$$

## 5. Trade-off curves

Relationships in two-dimensional space of such criterions as output and control signal variances determine the trade-off curve (Huang & Shah 1999) which separate two regions: achievable (above) and non-achievable (below). On the basis of point location with respect to the trade-off curve one can assess the control performance. Thus the trade-off curve can be defined by means the benchmark which minimizes quadratic performance index in the form of (9). This means that such benchmark acts the lower bound taking the control effort into account.

Since standard deviations better characterize signal magnitudes, in the chapter trade-off curves are drawn on the plane  $\text{std}(y)$ - $\text{std}(u)$ .

Therefore the LQG control benchmark for a linear continuous-time plant whose output is corrupted by a stochastic disturbance controlled by a discrete-time controller is proposed. The quality of control systems with PID controllers tuned both classically (QDR-Quarter Decay Ratio) and optimally in such way that disturbance characteristics are taken into account is investigated and, assuming the same control effort, compared with the benchmark. It has been shown that optimal tuning of classical PID controllers improves the disturbance attenuation bringing it closer to the lower bound.

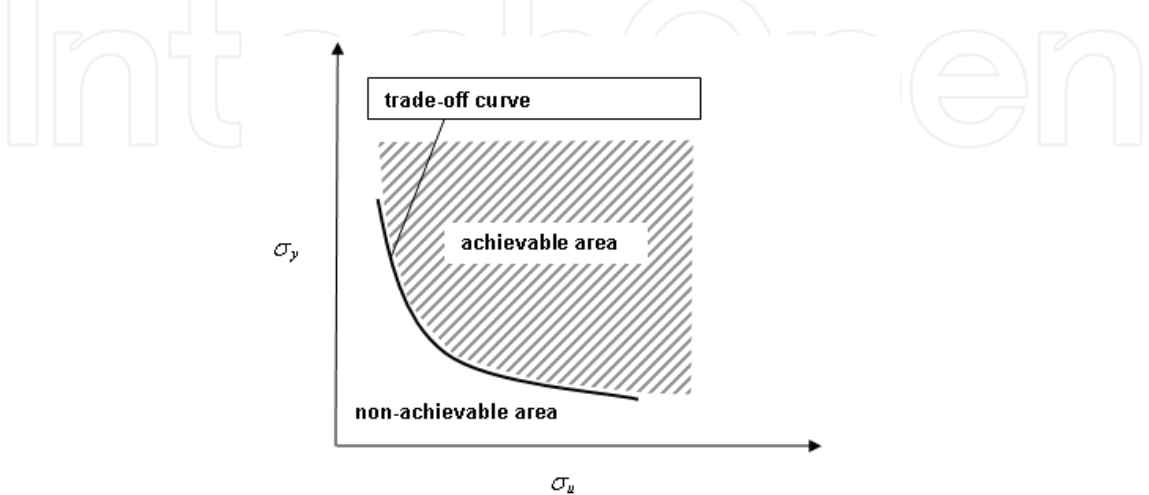


Fig. 1. Trade-off curve

In Fig. 1 standard deviation of output signal against standard deviation of control signal for systems with LQG and optimally tuned PID type controllers is plotted<sup>1</sup>. Optimal values of these parameters were received for varying values of the weighting factor  $\lambda$ . Results of the minimum variance strategy are plotted as horizontal lines. And results of PID type control with controllers settings supplied by means of QDR method are plotted as points.

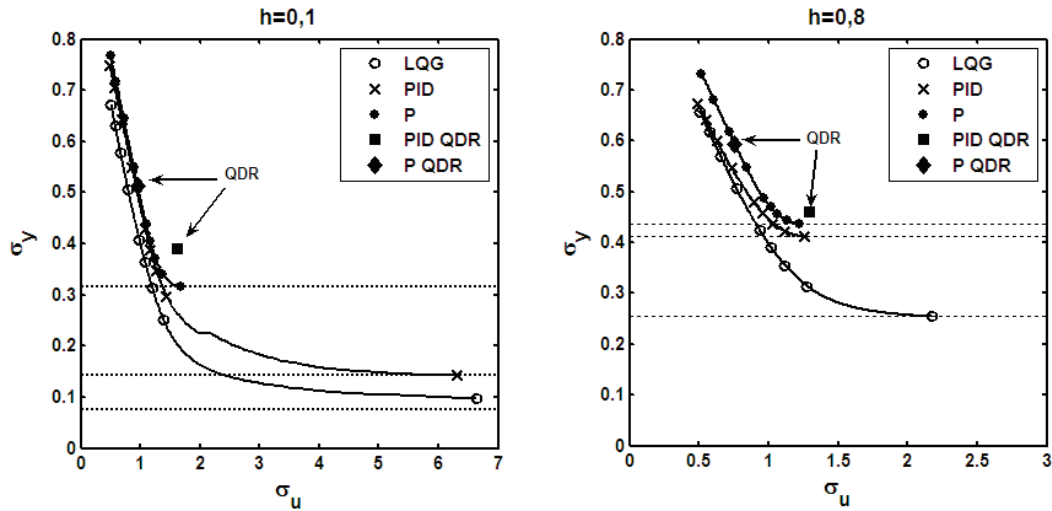


Fig. 2. Standard deviation of output vs control signal (trade-off curve)

<sup>1</sup> Exemplary plant transfer function:  $G^p(s)=\frac{I}{(\alpha s+I)(s+I)^2}$ ,



6. Control performance assessment based on lag-delay model

Suitability of control quality assessment based on delay approximation for delay-free plants is investigated in this section. To this end, the LQG control benchmark, which can be seen as a MV benchmark with bounded control variance, is compared for both linear delay-free continuous-time plants with outputs corrupted by a stochastic disturbance and their lag-delay models. Using approximated plant models, the area of achievable accuracy is then defined for control performance assessment. The transfer function  $G^P(s)$ <sup>2</sup> is then approximated by the lag-delay transfer function (3) $H^P(s)$ . Comparison of their step responses is given in Fig. 3.

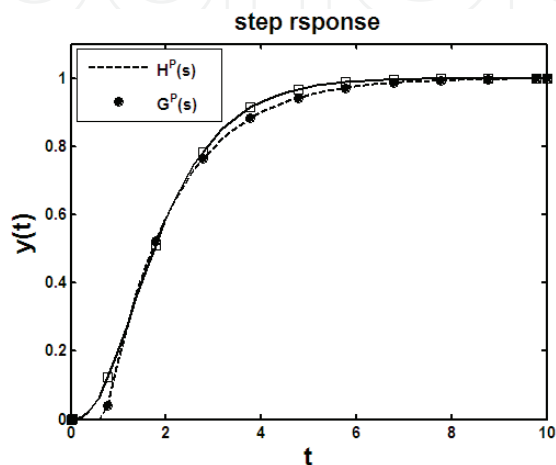


Fig. 3. Step response characteristics of original delay-free plant and its lag-delay model.

In Fig. 4 standard deviations of output signal against standard deviations of control signal for both delayed and non-delayed systems controlled by, respectively, optimal LQG and PID type controllers are plotted. The plots are parametrized by the weighting factor  $\lambda$ . Results of the MV ( $\lambda=0$ ) strategy are plotted as horizontal dashed lines and define areas of uncertainty of the performance lower bound when using the lag-delay approximation of non-delayed systems. Results of the benchmark designed for original model with realistic control signal magnitudes ( $\lambda=0.001$ ) and PID type control with controllers settings supplied by means of QDR methods are plotted as points.

The MV control strategy used for the delayed model gives lower control quality than the MV algorithm for the delay-free model. Furthermore there is no significant improvement of control quality when optimal settings for classical controllers are used for the delayed model as compared to those obtained by means of QDR method.

Since control signal magnitudes of the lower bound i.e. LQG ( $\lambda=0$ ) for lag-delay approximation (3) and those achieved with PID controller are comparable, the most popular MV benchmark  $\eta$  of (1) makes sense when using delayed models, indicating that relatively good performance. It can unfortunately hide the possibility of further performance improvement when delay-free model is used.

<sup>2</sup>  $G^P(s)=\frac{I}{(s+I)(0.5s+I)^2}$

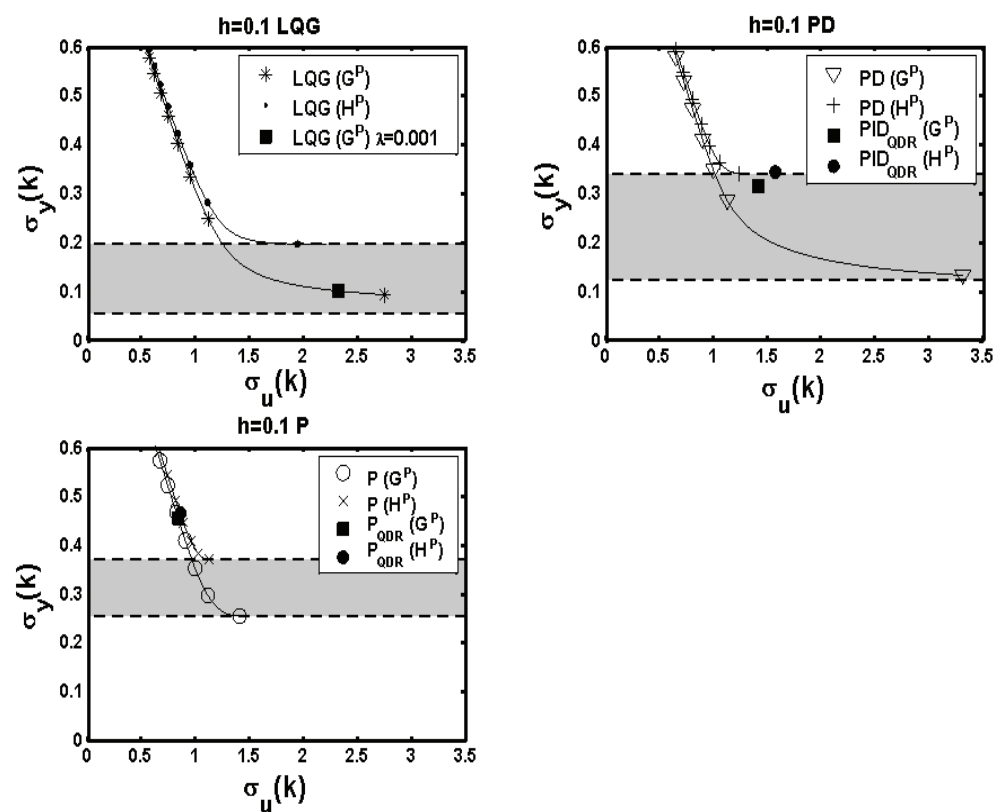


Fig. 4. Areas of achievable accuracy using lag-delay approximation.

Table 1 presents the values of index  $\eta$  and standard deviation of output signal  $\sigma_y$  for the systems with the plant transfer function  $G^P(s)$  and  $H^P(s)$ . Furthermore the value of  $\sqrt{\eta} \cdot \sigma_y$  is shown which is equivalent to standard deviation of the output signal obtained when pure MV Astrom’s algorithm is used.

$\lambda=0$ $h=0.1$		$\eta$	$\sigma_y$	$\sqrt{\eta}$	$\sqrt{\eta} \cdot \sigma_y$
LQG	$[G^P(s)]$	0.8421	0.0751	0.9176	0.0689
PD	$[G^P(s)]$	0.2668	0.1333	0.5166	0.0689
P	$[G^P(s)]$	0.0701	0.2601	0.2648	0.0689
PID <sub>QDR</sub>	$[G^P(s)]$	0.0480	0.3145	0.2088	0.0689
P <sub>QDR</sub>	$[G^P(s)]$	0.0230	0.4545	0.1517	0.0689
LQG	$[H^P(s)]$	0.9998	0.2026	0.9999	0.2026
PD	$[H^P(s)]$	0.3465	0.3442	0.5886	0.2026
P	$[H^P(s)]$	0.2938	0.3738	0.7362	0.2026
PID <sub>QDR</sub>	$[H^P(s)]$	0.3485	0.3432	0.5652	0.2026
P <sub>QDR</sub>	$[H^P(s)]$	0.1900	0.4649	0.4359	0.2026
substitute delay					
LQG	$[G^P(s)]$	1.0000	0.0751	1.0000	0.0751
PD	$[G^P(s)]$	0.4992	0.1333	0.7065	0.0942
P	$[G^P(s)]$	0.4315	0.2601	0.6568	0.1709
PID <sub>QDR</sub>	$[G^P(s)]$	0.2799	0.3145	0.5094	0.1664
P <sub>QDR</sub>	$[G^P(s)]$	0.1773	0.4545	0.2791	0.1914

Table 1. Values of performance measure  $\eta$  and estimated output standard deviation under pure MV Astrom’s control algorithm

The bottom part of the table shows the values calculated for the delay-free system,  $G^P(s)$ , and calculated using the estimated substitute delay. The most important conclusion is that the use of the substitute delay to assess the system control performance often gives unreliable results. Another interesting observation is that the values from the last column belong to the uncertainty area of performance lower bound (see Fig. 3). This can also be seen from Fig. 5s where the estimates of the performance lower bound are plotted when assuming different values of delay  $l$ .

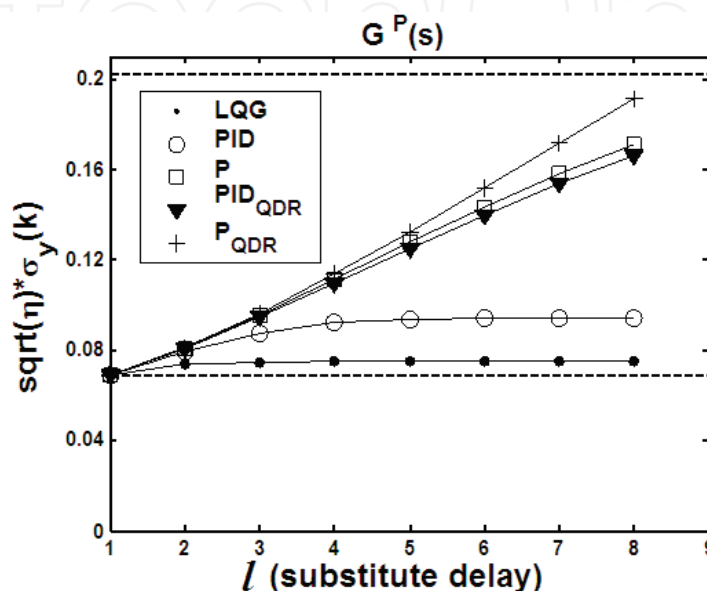


Fig. 5. Areas of achievable accuracy using lag-delay approximation.

## 7. More complete characteristic of control error

As mentioned in the previous section using more sophisticated original delay-free model additional improvement of control quality can be attained for both PID and LQG controllers. The price paid is much larger control variance. It is interesting to note that while LQG systems remain robust this is not longer valid for PID controllers.

Then in this section comparison of certain time and frequency domain functions will be done to give further insight into assessment of control performance in terms of system robustness.

The notion of 1D-PID will be also used to denote the limited authority tuning of PID controller whose dynamical parameters  $T_i$  and  $T_d$  are chosen from a popular tuning rule, e.g. the QDR method based on model (3), remain constant, and only the gain  $K_p$  is chosen so as to minimize the index in (9).

In Fig. 6. trade-off curves displaying standard deviations of output signals against standard deviations of control signals for the original delay-free system controlled by optimal LQG controller and by optimally tuned 1D-PID controller are plotted. Plots are parameterized by the weighting factor  $\lambda$ , and results of the unrestricted MV-LQG and restricted structure, 1D-PID MV strategies ( $\lambda=0$ ) are plotted as horizontal dotted lines for both types of controllers. Due to excessive control actions and small increase of control quality MV based benchmarks

are not particularly authoritative [3] for non-delayed systems. Therefore, a benchmark which assumes restricted control effort seems to be more suitable for systems controlled by the PID controllers. Points a→a1, b→b1, c→c1 depict correspondence of control systems with the same control effort.

	$\eta$	$\sqrt{\eta}$
a	0.4846	0,6961
b	0.3075	0,5545
c	0.3018	0,5494
a1	0.7932	0,8906
b1	0.7071	0,8409
c1	0.5853	0,7650

Table 2. Performance measure for systems in Fig. 6

It is worth noting that for the MV-LQG  $\eta=0.9321$  ( $\sqrt{\eta}=0,9655$ ). The almost MV-LQG system represented as a1 has the value of  $\eta=0.7932$  ( $\sqrt{\eta}=0,8906$ ), and for a reasonably tuned system represented by c1 there is  $\eta=0.5853$  ( $\sqrt{\eta}=0,7650$ ), respectively.

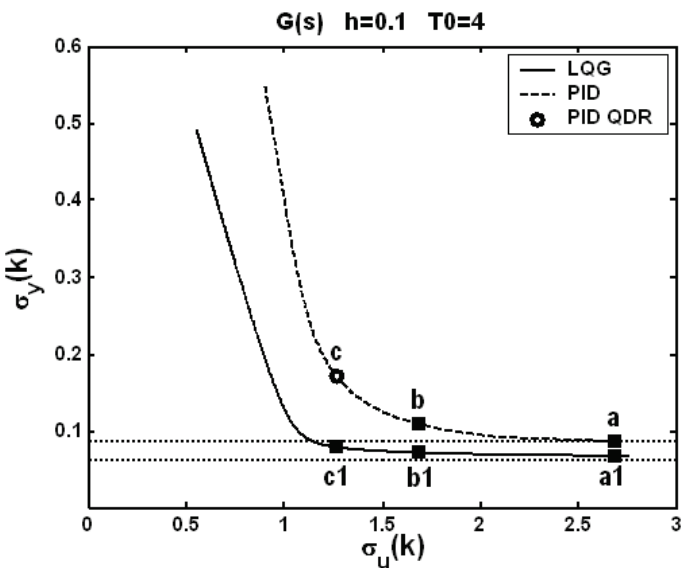


Fig. 6. Standard deviation of output vs control signal for PID and LQG controlled systems.

In Fig. 7-8 PSD<sup>3</sup> functions of output signals of corresponding systems are plotted and compared with the PSDF of disturbance. Important observation is that the optimal PID controller distinguishes itself by very poor robustness both in terms of phase margin and large sensitivity peak. This fact is reflected by the appearance of high frequency peak in PSD function. This is not the case for moderately tuned PID and all LQG controllers.

<sup>3</sup> PSD - Power Spectral Density function

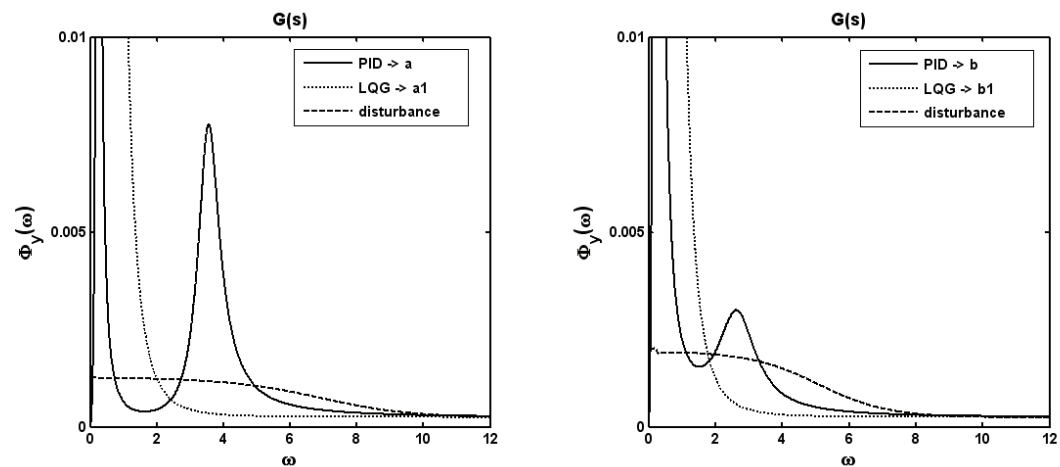


Fig. 7. PSDF of output signals for LQG and PID loop in comparison with PSDF of disturbance corresponding to a, a1, b, b1.

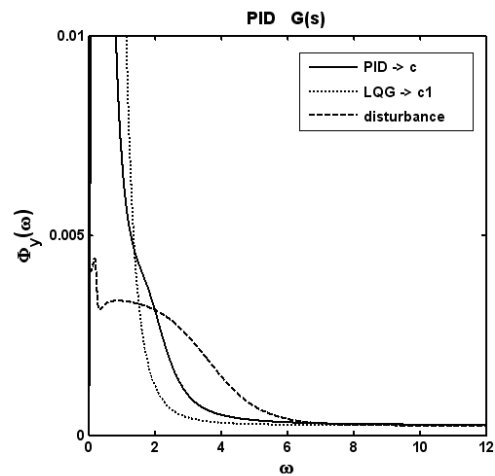


Fig. 8. PSDF of output signals for LQG and PID loop in comparison with PSDF of disturbance; corresponding to c, c1.

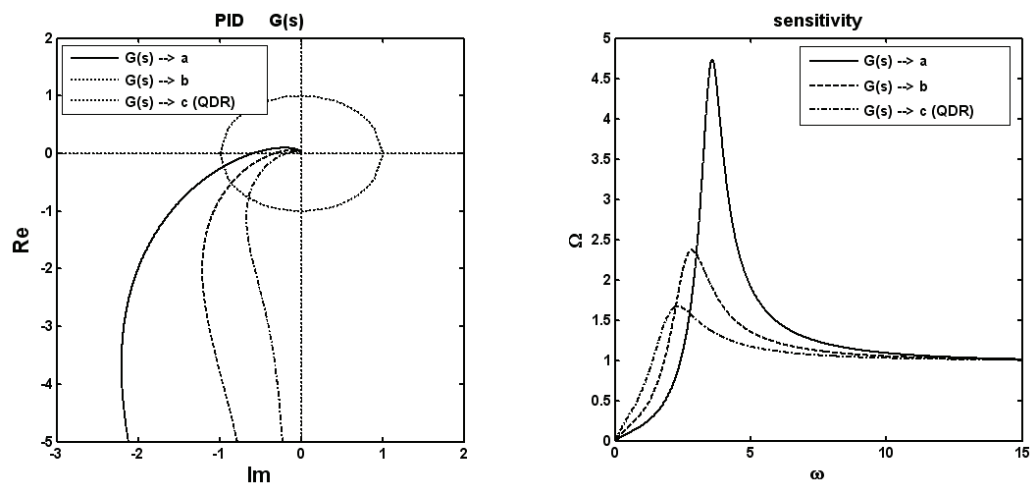


Fig. 9. Nyquist plots of the open loop and the sensitivity function for PID control corresponding to a, b, c.

In Fig. 9-11 Nyquist plots, step responses and sensitivity functions of systems corresponding to points a,b,c and a1, b1, c1 are plotted. The main outcome is that similar efficiency of disturbance attenuation can be attained with both MV-LQG and 1D-PID controllers. Unfortunately, in contrast to LQG, increasing control efficiency results in poorer robustness of 1D -PID MV control systems.

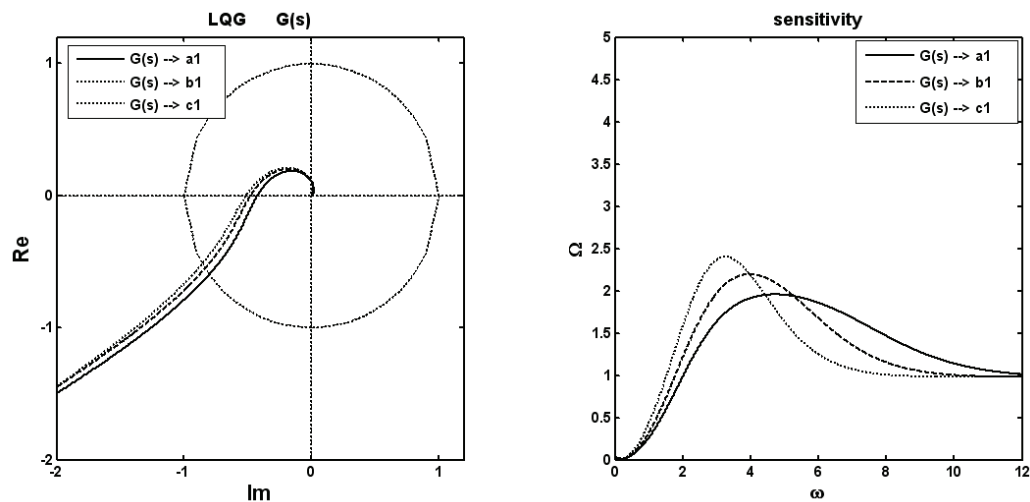


Fig. 10. Nyquist plots of the open loop and the sensitivity function for LQG control corresponding to a1, b1, c1.

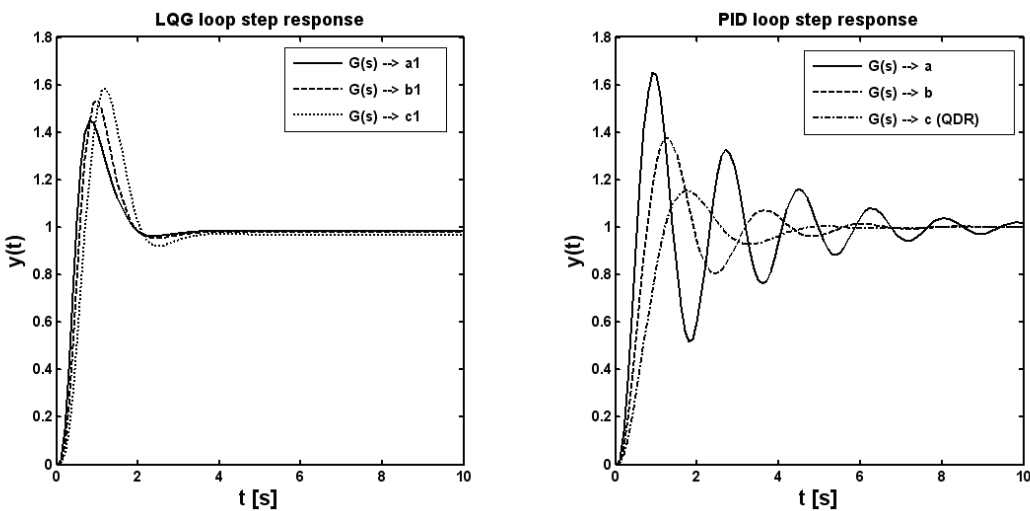


Fig. 11. Step response characteristics of LQG and PID loops corresponding to a, b, c and a1, b1, c1.

8. Remarks on single-stage against multi-stage performance criteria for control benchmarking

The performance measure which takes control effort into account becomes more and more popular. This measure result in solution of GMVC or LQG problem. And this next leads to the minimization of single-stage or computationally more complicated multi-stage quadratic

performance criteria. The section shows that simple single-stage performance criteria have limited application for unstable and non-minimumphase plants. Further it was shown that for sampled data systems integral performance criterion which takes inter-sample behaviour into account is more suitable.

Single-stage and infinite-horizon performance criteria will be compared due to their use for control system benchmarking. It will be shown that when using simple single-stage cost function, a critical value of control weighting might exist, under which the control system loses its stability. As far as the infinite horizon is concerned, this problem does not exist. Furthermore, if uncontrolled system has unstable discretization zeros, then even a single step cost function can assure stability of the closed loop, provided that the single stage discrete-time performance index is produced by integrating a continuous time index within the sampling period.

To compare the system behaviour under single-stage in contrast to multi-stage criteria standard deviations of output and control signal in sampling instances will be used again. These parameters describe signal magnitudes better than variances and are also useful to assess the control performance.

If the uncontrolled system is unstable or non-minimumphase, controller designed by means of single-stage criterion can be also unstable. This was presented in Fig. 12 and Fig. 13 by means of values of standard deviations of output and control signals.

The exemplary uncontrolled first-order unstable system is given by following transfer function:

$$G_1(s) = \frac{1}{(s-1)} \quad (38)$$

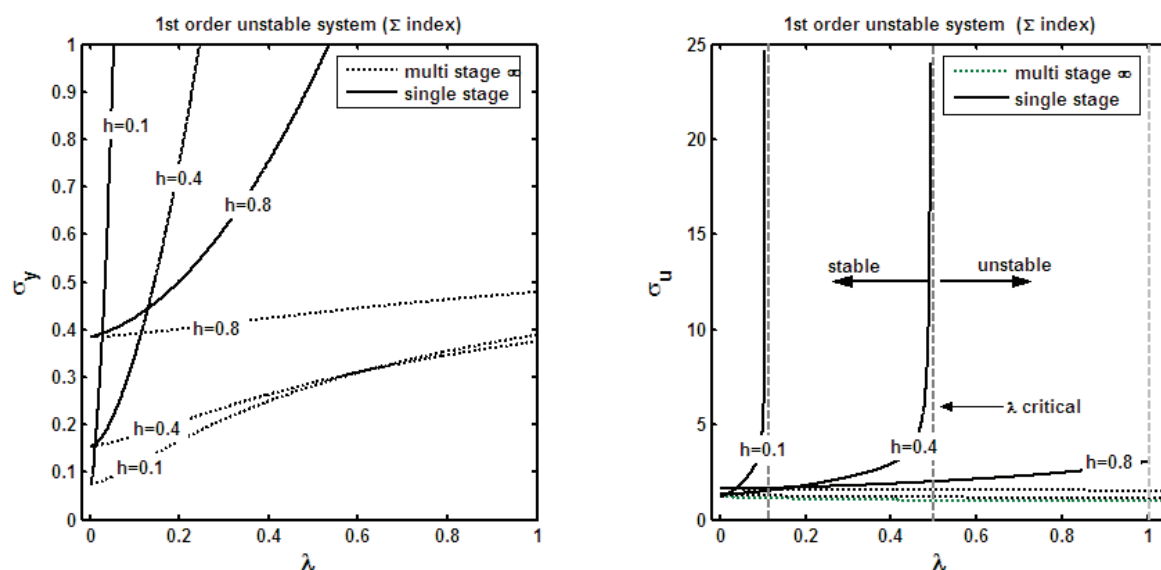


Fig. 12. Standard deviations of output and control signals against weighting factor  $\lambda$  for both single-stage and multi-stage criteria.

Vertical dashed lines represent critical values of weighting factor when the closed loop system controlled by algorithm based on single-stage criteria is unstable. This is not valid for more complicated control algorithm which minimizes multi-stage performance function. Fig. 13 represents results for an exemplary non-minimumphase plant described by following transfer function:

$$G_2(s)=\frac{I}{(s+1)}\left(\frac{1-\alpha s}{1+\alpha s}\right), \quad \alpha=0.2$$

(39)

The closed loop system with controller designed by means of optimization multi-stage performance index remains stable for all values of  $\lambda$  in contrast to this simple single-stage index.

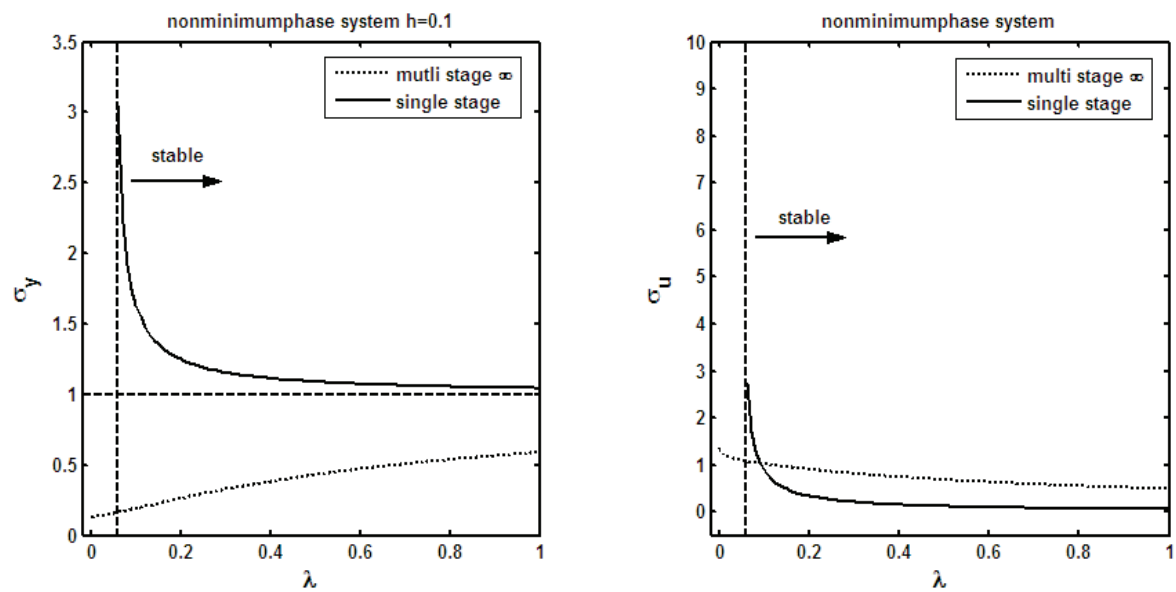


Fig. 12. Standard deviations of output and control signals against weighting factor  $\lambda$  for both single-stage and multi-stage criteria.

8.1 Integral cost function

Next figures illustrate using discrete versus integral cost function for the plant given by following transfer function

$$G_3(s)=\frac{I}{(s+1)}e^{-s\tau} \quad \tau=0.7 \quad h=0.8$$

(40)

Discretization of the transfer function  $G_3(s)$  with period  $h$  gives unstable zeros. In contrast to the discrete cost function integral criterion both multi-stage and single-stage provide stable results for the closed loop system.



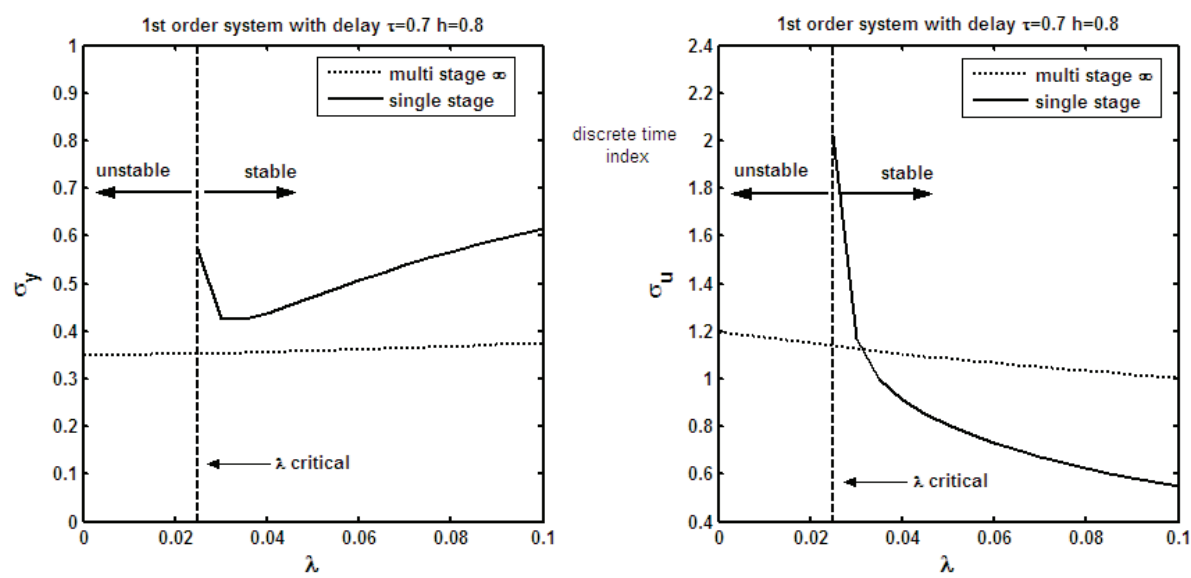


Fig. 13. Standard deviations of output and control signals against weighting factor  $\lambda$  for both single-stage and multi-stage criteria (discrete index).

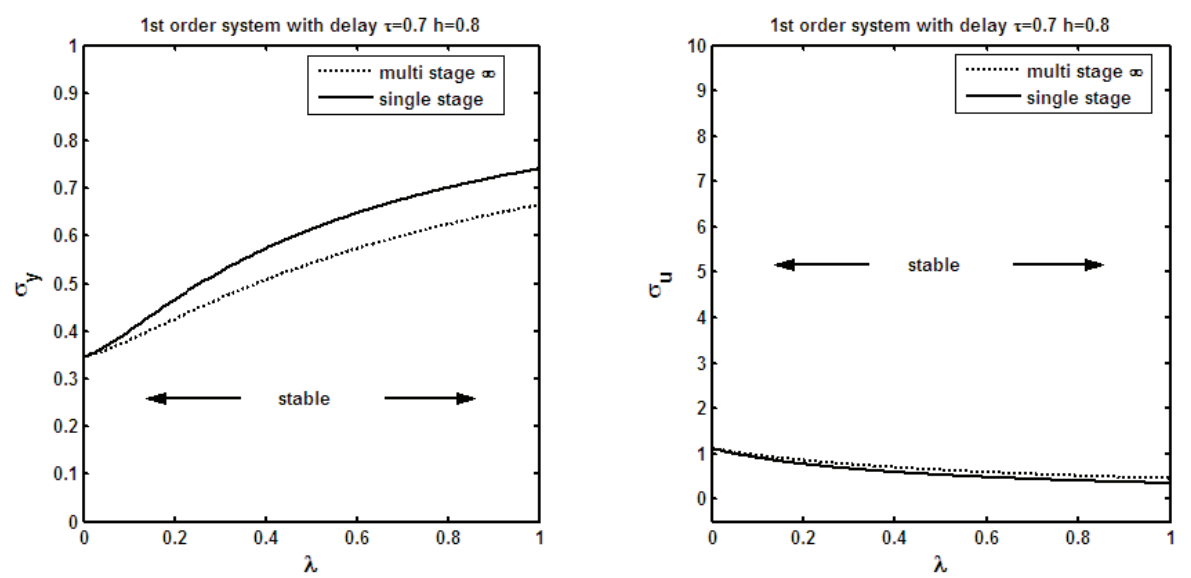


Fig. 14. Standard deviations of output and control signals against weighting factor  $\lambda$  for both single-stage and multi-stage criteria (integral index).

9. Conclusion

In the chapter some developments in the control performance assessment are provided. The solution based on quadratic performance criteria which taking control effort into account was proposed in return for popular MV measure. This further broke about the definition of trade-off curve using standard deviation of both control and error signals. The standard deviation parameter is preferred because better than variance characterize the signal

magnitudes. The tool in the form of trade of curve showed to be very useful for the control quality assessment of systems equipped with restricted structure controllers, such as PID type. The area of achievable accuracy, when models with substitute delay is used, was also defined showing the possibility of further improvement of control performance when more sophisticated non-delayed model of plant is applied. The price paid is much larger control variance and loss of robustness in case of the system with PID type controllers. Therefore from the technological point of view the knowledge of control error variance only is not sufficient enough and more complete characteristic which gives further insight into control performance assessment in term of robustness is necessary. In the end the problem of using simple single-stage and computationally more complicated multi-stage quadratic performance criteria was exemplified. That was pointed out existing of the critical value of weighting factor which result in an unstable controller design if uncontrolled system is unstable or non-minimumphase and the simple single-stage cost function is used.

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The book *New Approaches in Automation and Robotics* offers in 22 chapters a collection of recent developments in automation, robotics as well as control theory. It is dedicated to researchers in science and industry, students, and practicing engineers, who wish to update and enhance their knowledge on modern methods and innovative applications. The authors and editor of this book wish to motivate people, especially under-graduate students, to get involved with the interesting field of robotics and mechatronics. We hope that the ideas and concepts presented in this book are useful for your own work and could contribute to problem solving in similar applications as well. It is clear, however, that the wide area of automation and robotics can only be highlighted at several spots but not completely covered by a single book.

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